HYDRODYNAMICS OF A GRANULAR MEDIUM

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The hydrodynamic characteristics of loose granular media are studied experimentally, with due allowance for particular aspects of the pore space structure (the sinuous nature of the paths of penetration and the efficiency with which the intergranular spaces take part in filtration); the results show that $\lambda \, Re \approx 80$, up to Reynolds numbers of $Re_{cr} \approx 5$.

Consideration of the fundamental laws of hydrodynamics – the Navier–Stokes equations – shows that the energy loss (pressure drop Δp) in a steady-state flow is determined by the forces of internal friction (viscosity) and the inertial forces, or $\Delta p = \Delta p_v + \Delta p_i$. When the viscous forces predominate Δp_i is negligibly small; hence any increase in pressure is accompanied by a directly proportional increase in the rate of flow W; this occurs, for example, in the flow of liquids or gases through tubes. This form of flow is reflected in the well-known expression for the coefficient of local resistance λ in terms of the Re number, which takes the form

$$\lambda = 64/Re \tag{1}$$

in the range Re < 2300. However, opinions differ when filtration through a porous medium is considered. Some research workers [1, 2] assume that the conditions of filtration in a porous medium are reflected by the expression

$$\lambda = C/Re, \tag{2}$$

where C is a constant number differing from 64 as a result of the curvature of the channels in the porous medium, noncylindrical cross section of these channels, and the influence of the expansion and contraction of the channels on their permeability.

Other research workers [3] consider that the conditions of filtration are more accurately represented by a two-term expression of the form

$$\lambda = C/Re + B, \tag{3}$$

in which the second term reflects the influence of the inertial forces and is quite perceptible for even the lowest filtration rates. The majority of workers adopt an intermediate position in this respect, but their many determinations of Re_{cr} yield very different results [4-12]. This is because, in a granular medium, the transition from the laminar to the turbulent mode of flow takes place in a much smoother manner than in tubes. The reasons are as follows. Firstly, there are a large number of channels with differing cross sections in a granular medium; secondly, consideration of the filtration process as an "external" hydrodynamical problem indicates that, when a flow passes around the particles, the inertial terms determining the nonlinearity of the flow situation tend gradually to zero with diminishing velocity, rather than vanishing suddenly when a critical velocity is reached. Hence, when studying a granular medium, we may only speak of a "practical" value of the average Reynolds number, up to which the resistance law may be regarded as approximately linear.

Analysis of the experimental data in [13, 14] shows that the $\Delta p/LW = f(W)$ relationship may be represented by a broken straight line (which is obtained repeatedly under different experimental conditions), as illustrated in Fig. 1. The results presented in Fig. 1 suggest that, in a granular medium, as in a tubular conduit, the filtrate moves in a viscous manner at low filtration velocities, with hardly any inertial forces.

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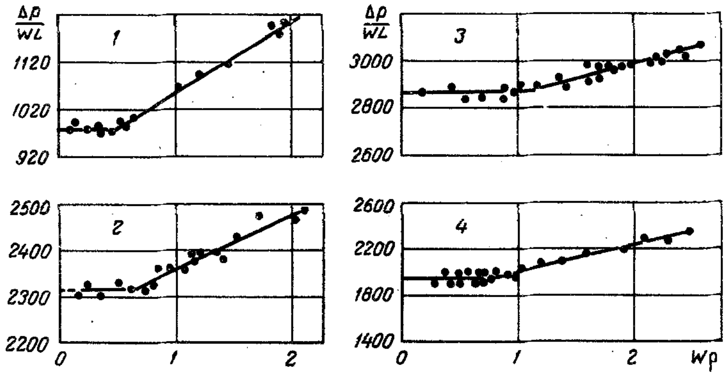

Fig. 1. Experimental data [13, 14] for shot charges of the following dimensions: 1) 1.6-1.5 mm (filtrate kerosene, experiment No. 41); 2) 1060-900 μ (filtrate kerosene, experiment No. 42); 3) 1.6 mm (filtrate kerosene–oil mixture); 4) 1.4 mm (filtrate kerosene). Cases 1 and 2 refer to [14], cases 3 and 4 to [13].

This mode of filtration is represented by the horizontal straight lines in Fig. 1; these only transform into inclined lines after reaching a certain critical velocity W_{cr} – at the break in the straight line. Thus for $W < W_{cr}$ an expression of type (2) should be valid, and for $W > W_{cr}$, when the effect of the inertial forces becomes appreciable, an expression of type (3).

In order to estimate the quantities C, Re, and λ, we used the apparatus illustrated schematically in Fig. 2. After first weighing and determining the specific gravity, the test layer of loose material was poured into the transparent cylindrical part of column 1. The inside area of the column was $A = 100\ cm^2$. We measured the height of the layer L and calculated the intergranular porosity ε. Then we passed air through the layer by means of the air blower 2 and receiver 3, measuring the filtration velocity W with a gas counter 4 and the loss of head Δp with a manometer 5. After measuring Δp for various W, an electrode 7 was placed on the upper plane of the layer and the column was slowly filled from the bottom with an electrolyte passing through the funnel 8. For this purpose a vacuum was first established in the column. The electrodes 7 were connected to a four-armed ac measuring bridge working at a frequency of 5000 cps. In this way we measured the electrical resistance R_l of the layer consisting of nonconducting particles, the intergranular pores of which were filled with the electrolyte. In order to determine R_{el}, the material was removed from the column, the electrodes were placed as before, and the column was filled with the electrolyte only, the resistance of this being measured at the same temperature as before. As subjects for

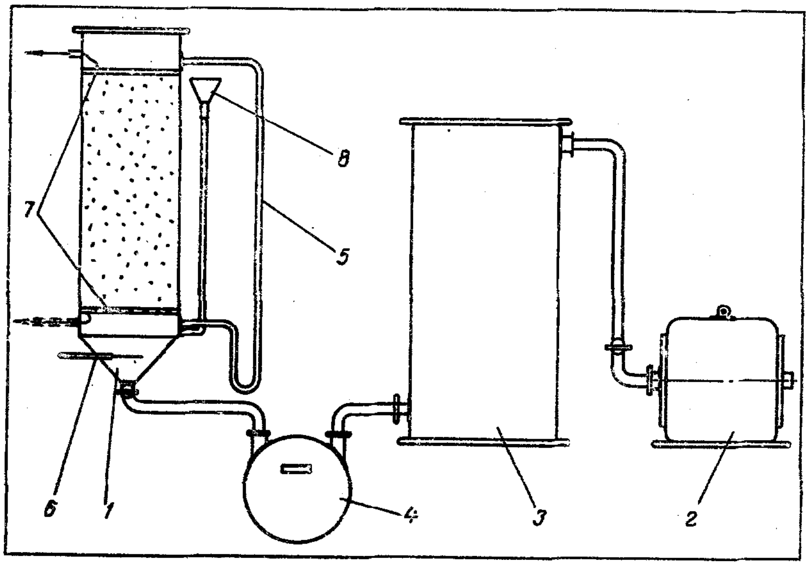

Fig. 2. Arrangement of the experimental apparatus: 1) column; 2) air blower; 3) receiver; 4) gas counter; 5) differential manometer; 6) mercury thermometer; 7) electrodes (grids); 8) funnel.

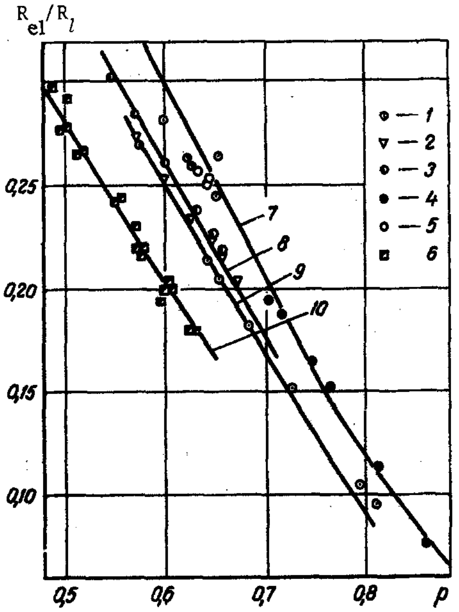

Fig. 3. Dependence of the relative electrical permeability R_{el}/R_l on the compaction of the charge p: 1) cubes; 2) cylinders; 3) disks; 4) spheres of many sizes; 5) spheres of a single size; 6) crushed marble (1-5 from [15]; 6, authors' own results); 7) φ = 0.5; 8) 0.8; 9) 0.9; 10) 1.5.

study we chose loose charges formed by particles of regular geometrical shapes (spheres, cylinders, cubes, and disks), and also particles of irregular shape obtained by crushing marble and then classifying it by means of sieves. The results of our determinations of R_{el}/R_l for various ε are illustrated graphically in Fig. 3, which also shows some points taken from [15]. A mathematical description of the relationship $R_{el}/R_l = f(P)$, where $|P = 1-\varepsilon|$, was proposed by Rayleigh for cubically packed spheres [16]:

$$\frac{R_{el}}{R_l} = 1 - \frac{3P}{2 + P - 0.392P^{10/3}}. \tag{4}$$

Maxwell [17] proposed another expression for spheres

$$\frac{R_{el}}{R_l} = \frac{1-P}{1+0.5P}. \tag{5}$$

In Eq. (4) the character of the cubic arrangement is reflected in the quantity $0.392\,P^{10/3}$. If we neglect this, (4) transforms into (5).

For particles of irregular shape Fricke proposed the introduction of a form factor φ into (5); for spheres this should be equal to 0.5. Then according to Fricke

$$\frac{R_{el}}{R_l} = \frac{1-P}{1+\varphi P} = \frac{\varepsilon}{1+\varphi P}. \tag{6}$$

When the shape of the particles deviates from spherical φ increases [19, 20]. However, Eq. (6) fails to reflect the effect of the arrangement of the particles on the relative electrical permeability $|R_{el}/R_l|$, as does (4). Allowing for all the foregoing considerations, for a random arrangement of irregular particles we introduced a form factor φ into (4), with the empirical value $0.2\,P^{10/3}$ instead of the analytical $0.393\,P^{10/3}$. Then for random charges of particles of any shape

$$\frac{R_{el}}{R_l} = 1 - \frac{3P}{2 + \varphi P - 0.2P^{10/3}}. \tag{7}$$

We see from Fig. 3 that for different φ Eq. (7) agrees satisfactorily with experimental data over a wide range of values of p.

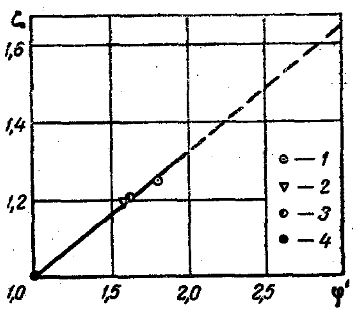

Fig. 4. Dependence of the surface coefficient ζ on the relative form factor of the particles φ': 1) cubes; 2) cylinders; 3) disks; 4) spheres.

TABLE 1. Initial Data and Results of Our Determinations of Re_{cr} and λRe

No.	Shape	Material	Dimensions, mm	ε	L, cm	S, cm	ε_{eff}	L_{sin}/L	Filtrate	W_{cr}, cm/sec	Re_{cr}	λRe
1	Spheres	Lead	6,10	0,370	29,0	9,8	0,312	1,08	Air	1,29	5,36	75,4
2		Glass	2,63	0,387	25,0	22,8	0,329			2,33	4,26	73,8
3		Peas	5,93	0,369	30,0	10,1	0,311			1,34	5,40	78,5
4				0,383	50,0	10,1	0,325	1,07		1,30	5,32	85,8
5		Polystyrene	3,78	0,397	24,5	15,8	0,340			1,92	5,12	76,4
6		Glass	2,30	0,379	27,2	26,1	0,321			2,67	4,20	83,0
7		Polystyrene	5,4—1,9	0,385	55,0	22,8	0,327			2,92	5,30	71,0
8		Lead	1,60	0,364	45,0	37,5	0,306	1,08	Kerosene	0,84	4,58	66,5
9				0,364		37,5	0,306			1,10	3,38	67,0
10			1,40	0,359		42,7	0,301			0,966	4,68	68,7
11			1,20	0,354		50,0	0,297			2,70	5,55	70,9
12										1,14	4,58	72,4
13			0,95	0,350		63,1	0,293			1,66	4,88	68,2
14	Cylinder	Glass	∅3,8×7,4	0,390	21,8	13,2	0,306		Air	1,67	5,21	79,5
15	Disk	Polystyrene	∅2,7×4,6	0,396	54,5	19,4	0,312	1,10		2,50	5,37	78,3
16	Disk		∅3,9×2,0	0,399	56,3	20,4	0,315			3,00	6,16	72,1
17	Disk	Polyethyl	3,98	0,412	54,0	15,1	0,319	1,12		2,08	5,94	70,4
18	Irregular	Marble	3,00	0,479	33,0	32,8	0,340			3,58	4,90	77,7
19			4,50	0,510	54,5	21,8	0,370			3,33	7,15	72,8
20			6,00	0,499	11,5	14,0	0,359			2,17	7,08	77,8
21				0,503	46,0	13,1	0,363	1,13		2,03	7,12	80,5
22				0,481	40,0	16,4	0,343			2,67	7,35	82,7
23			8,50	0,512	19,3	11,6	0,372			1,68	6,80	76,0
24				0,494	25,0	11,6	0,355			1,71	6,75	77,6
25			3÷12,5	0,389	41,5	26,4	0,260	1,15		4,17	6,50	74,5
26				0,403	20,0	27,5	0,272			2,75	4,13	79,3

Note: 1. Initial data Nos. 8-13 taken from [13]. 2. In experiment No. 11 the viscosity of the filtrate $\mu = 5.05 \cdot 10^{-2}$; in No. 12 $2.59 \cdot 10^{-2}$ P.

From the value of φ we may also define a surface coefficient ζ for bodies of regular geometrical shape. This coefficient equals the ratio of the actual specific surface area of the body S_{tru} to the nominal specific surface area S_{nom} calculated from the volume of the body on the assumption that it has a spherical shape.

Clearly for a sphere this ratio (or surface coefficient ζ) is equal to unity. For all other shapes the ratio should be greater than unity. Let us now introduce a relative particle form factor φ', for all these shapes of body, indicating how many times the φ of the shape under consideration exceeds the φ of a sphere. We may generalize all this as follows:

Shape	Sphere	Cylinder	Disk	Cube
φ	0.5	0.8	0.8	0.9
$\varphi' = \varphi/0.5$	1.0	1.6	1.6	1.8
ζ	1.0	1.2	1.2	1.245

On the basis of these data we may plot the graph illustrated in Fig. 4, which indicates that the $\zeta = f(\varphi')$ relationship is in fact linear up to $\varphi' = 1.8$. If we subsequently assume the linearity of this relationship, we may determine ζ for any other shapes of particle by determining their experimental value of φ. It is then quite easy to calculate the specific surface area for these particles

$$S = \frac{6}{d} \cdot \zeta. \tag{8}$$

This method does not depend on the hydrodynamics of the layer, which is an indispensable condition in establishing hydrodynamic laws.

The results of our investigations into the relative electrical permeability of various charges illustrated graphically in Fig. 3 also enables us to calculate the effective intergranular porosity ε_{eff} and the sinuosity (tortuosity) L_{sin}/L of the paths of penetration from the expressions proposed earlier [21]:

$$\varepsilon_{eff} = \frac{(\varepsilon + R_{el}/R_l\,)\,2}{\left(1 + \dfrac{\varepsilon}{R_{el}/R_l}\right) + \left(1 + \dfrac{R_{el}/R_l}{\varepsilon}\right)}, \tag{9}$$

$$L_{sin}/L = \sqrt{(R_l\,/R_{el})\,\varepsilon_{eff}}\,. \tag{10}$$

In order to reflect the hydrodynamics of the granular layer, we make use of the following expression, well known in hydraulics:

$$W_{tru} = \frac{\Delta p R_{hyd}^2}{k_0 L \mu}.$$

For different shapes of channel we have [12] the following data:

Shape of cross section	Circle	Equilateral triangle	Square	Slit $a = 10b$
Channel form factor k_0	2.00	1.67	1.78	2.65

For a sinuous channel we may write

$$W_{tru} = \frac{\Delta p R_{hyd}^2}{k_0 \mu L_{sin}}, \quad \text{or} \quad W_{tru} = \frac{\Delta p D_{eq}^2}{16 k_0 \mu L_{sin}}. \tag{11}$$

$$\mathrm{Re} = \frac{W_{tru} D_{eq} \rho}{\mu}, \tag{12}$$

$$\Delta p = \lambda \frac{W_{tru}^2 \rho L_{sin}}{2 D_{eq}}. \tag{13}$$

If we find μ from (12), substitute it into (11), and carry out certain transformations, we obtain

$$\Delta p = \frac{k_0 32 W_{tru}^2 \rho L_{sin}}{\mathrm{Re} 2 D_{eq}}. \tag{14}$$

Comparing (14) with (13) and taking $k_0 \approx 2.5$ for the granular layer we have

$$\lambda = \frac{k_0 32}{\mathrm{Re}} \approx \frac{80}{\mathrm{Re}}. \tag{15}$$

We write

$$W_{tru} = \frac{W}{\varepsilon_{eff}} \cdot \frac{L_{sin}}{L} \text{ and } D_{eq} = \frac{4\varepsilon_{eff}}{S(1-\varepsilon_{eff})}.$$

After substituting these values in (12) and (14), we obtain

$$\mathrm{Re} = \frac{4 W L_{sin} \rho}{S(1-\varepsilon_{eff}) L \mu}, \tag{16}$$

$$\lambda = \frac{8 \Delta p \varepsilon_{eff}^3}{S(1-\varepsilon_{eff}) \rho L W^2} \cdot \left(\frac{L}{L_{sin}}\right)^3. \tag{17}$$

We carried out a series of experiments in the apparatus of Fig. 2 with bodies of regular geometrical shapes in the column; the hydraulic measurements gave reliable values of $W\Delta p$ and S, and the electrical conductivity measurements enabled us to estimate the values of R_{el}/R_l for known ε, so that ε_{eff} and L_{sin}/L could be deduced from (9) and (10).

The viscosity and density of air were taken from tables. From the $\Delta p/LW = f(W)$ graphs we determined the critical velocity W_{cr} at which the inertial forces became perceptible. For this velocity we calculated Re_{cr} from (16) and λ from (17). The calculation was carried out in an analogous manner for bodies of irregular shape (crushed marble), the only difference being that S was determined from (8). For marble $\varphi = 1.5$; $\varphi' = 3$ and according to Fig. 4 $\xi = 1.64$.

The initial data and the results are presented in Table 1. We see from Table 1 that, independently of the shape of the body, the dimensions and packing density of the particles, and the nature of the filtrate, Re_{cr} remained practically constant in all our experiments, and equal to ≈ 5, while $\lambda \mathrm{Re}$ was close to $C \approx 80$.

NOTATION

λ	is the local resistance coefficient;
Re	is the Reynolds number;
C, B	are the constants;
D_{eq}	is the equivalent channel diameter ($D_{eq} = 4R_{hyd}$, where R_{hyd} is the hydraulic radius);
d	is the diameter of the particles or the size of the mesh;
ε	is the porosity (ratio of the volume of the intergranular spaces to the whole volume of the layer);
P	is the particle packing density ($P = 1 - \varepsilon$);
S	is the specific surface area (ratio of the surface area of the particles to their volume);

Δp is the pressure drop;
W_{tru} is the velocity of flow in the channel;
μ is the viscosity of the filtrate;
L is the height of the particle charge;
R_{el} is the electrical resistance of the electrolyte filling the column;
R_l is the electrical resistance of the charge of nonconducting particles when the interparticle space is filled with electrolyte;
φ is the particle form factor (for spheres $\varphi = 0.5$);
φ' is the relative particle form factor ($\varphi' = \varphi/0.5$);
ρ is the density;
W is the rate of flow, referred to the whole cross section of the layer.

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